Introduction

The concept of algebraic expressions is something which provides difficulties for both teachers and students, as identified by Watson, Jones and Pratt (2013). This assignment considers how students struggle with understanding algebraic reasoning, focusing on the difficulties students have with “the nature of [mathematical] ‘answers’” (Booth 1988, p.21). Students are often uncomfortable with ‘unclosed’ answers (e.g. $a + b$) and like to join terms – possibly “reflect[ing] expectations derived from arithmetic concerning what “well-formed answers” (Matz 1980) are supposed to look like” (Booth 1988, p.24). This is what is known as the ‘lack of closure’ dilemma, where students are driven to conjoining terms in order to form an answer which is ‘closed’ rather than providing an ‘unclosed’ answer.

Part one: The concept, the difficulties and the research

Watson et al. (2013) noticed younger students generally have a basic understanding of relations between quantities; gained during primary education. However, they may not explicitly recognise rules which formed when generalising with algebra. For example, they may know that if $4 \times 3 = 12$, then $3 \times 4 = 12$, $12 = 3 \times 4$ and $12 = 4 \times 3$. However, understanding that $ab = c$ implies that $ba = c$, $c = ab$ and $c = ba$ progresses from knowing that this relation is true for all cases. This prior knowledge helps with understanding why equivalent terms can be collected when manipulating an algebraic expression and why there can be a ‘lack of closure’ with some algebraic solutions. A student may know that if $4 + 4 + 4 + 5$, collecting the $4$’s to get $(3 \times 4) + 5$ would help simplify an expression like $a + a + a + b$ and may also help in understanding why this kind of expression cannot be closed.

Having the ability to apply algebraic methods merely reflects an understanding of how to apply these methods rather than the underlying reasons – Skemp (1976) identifies this as ‘instrumental understanding’. I believe that I gained instrumental understanding at school; being taught to apply a method to answer questions without questioning why delayed my confidence in providing unfinished answers. Many of my classmates found conjoining terms (e.g. $3x + 2y = 6xy$) made sense, whereas I was often unhappy with lack of closure in the answer; however I did not feel compelled to ‘create’ answers. Skemp (1976) played devil’s advocate to think about reasons why instrumental understanding has advantages, however he immediately refutes these ideas – “nothing else but relational understanding can ever be adequate for a teacher” (Skemp 1976). I believe I can now fluently use letters as variables, manipulate expressions and use algebra in a variety of contexts giving me what Skemp identifies as a ‘relational understanding’. This relational understanding is how I define an understanding of algebra in pupils.

Several difficulties accompany learning algebra and progressing towards relational understanding. Watson et al. (2013) list common misunderstandings of notation made by students (originally identified by Küchemann 1981 and Booth 1984). This includes problems such as: ignoring letters in expressions (e.g. taking $3a$ as $3$), believing different letters have different values (e.g. not
believing the solution $x = 1, y = 1$ for $3x + 5y = 8$) and confusion of algebraic and arithmetic notation (e.g. $2 \times x$ is written as $2x$, but $2 \times 7$ is not 27).

In their review, Watson et al. (2013) found that grasping the two meanings of the equals sign significantly helps students to understand the relational meanings of algebra. It is quite easy to understand the concept of, ‘being equivalent or identical to’ (e.g. $4 + 3 = 7$ or $3(x + 1) = 3x + 3$), however algebraically the sign can also mean ‘is equal for some value of $x$’ (e.g. $2(x + 1) = 3x + 1$). Understanding generalised rules of addition helps with noticing these dual meanings (Jones 2008). In trials he found that students often dive straight into the question without checking the validity of statements. In order to really gain an understanding of algebra Usiskin (1988) states that students must first be able to identify the different uses. He considers the following equations:

1. $A = LW$
2. $40 = 5x$
3. $\sin x = \cos x \cdot \tan x$
4. $1 = n \cdot (1/n)$
5. $y = kx$

(Usiskin 1988, p.9)

“We usually call (1) a formula, (2) an equation (or open sentence) to solve, (3) an identity, (4) a property, and (5) an equation of a function of direct variation (not to be solved)” (Usiskin 1988, p.9). Each equation is viewed differently by someone who understands school algebra and without knowing whether there is a need to connect quantities, find unknowns, and consider arguments or generalisations, actually understanding these equations would be quite difficult. Confusion could even arise for someone with a basic grasp of algebra since the letters used in these equations could be interchangeable variables.

Herscovics and Linchevski (1994) noticed from Carpenter et al. (1981) that students found solving equations like $4 \times \square = 24$ much easier than concatenated equations such as $6m = 36$. This shows that some students have difficulty in understanding algebraic notation, in particular the concatenation. Using a letter, rather than an empty box, may also have caused confusion, since there is a common belief that a letter must mean something, whereas an empty box must be filled.

The ‘lack of closure’ dilemma is a major problem in learning algebra, as highlighted by Watson et al. (2013). A student may, for example, write $3x + 2y = 5xy$, which is incorrect, though systematically found. Clearly to a student without understanding of algebraic rules, conjoining terms here may make sense. Watson et al. (2013) identified from Tirosh et al. (1998) that this may be because of different approaches to teaching algebraic manipulation techniques. Some tutors try to encourage fluency in algebra, using lots of examples for practice and generalisation of procedures. Others try to encourage understanding using a conflict approach, bringing in substitution to compare how these procedures work in other orders and combinations. Tirosh et al. (1998) found the conflict based approach emphasised the ‘reason’ for manipulation rules. This issue of methodology of teaching algebra only to give students the skills to manipulate algebra, rather than an underlying relational understanding was also raised by Usiskin (1988).
Küchemann (1981) devised the algebra test for the mathematics part of the research programme ‘Concepts in Secondary Mathematics and Science’ (CSMS) in 1976. The test was designed to see students’ interpretation of notation and their levels of understanding in algebra. He identified that students’ interpret letters used in algebra in many ways. Hodgén et al. (2009) conducted the same test in 2008 to see how pupils understanding of algebra had developed and found the ‘lack of closure’ dilemma was still prominent. One question posed was “Add 4 onto 3n” where several pupils gave the answer 7n or just 7, clearly unable or unwilling to provide a solution that looks unfinished. It shows that children are often not satisfied with unclosed algebraic solutions and are driven to ignore important parts of questions to form closed answers. The question “If \( e + f = 8, e + f + g = \ldots \)” also featured in the test. Küchemann (1981) noticed that student tried to find numerical answers (e.g. \( g \) is the seventh letter of the alphabet so \( 8 + g = 15 \)) rather than accept an unclosed algebraic solution. There were also students who answered \( 8g \), again illustrating the issue of conjoining terms.

Stacey and MacGregor (1994) discuss how conjoining terms is a serious issue with learning algebra. They note that in the CSMS study students who did this were generally at the lowest level of understanding. Contextually, when learning to work with algebraic symbols students often relate to other learning experiences, for example “writing fractions […] represents addition (e.g. \( 3 \frac{1}{2} = 3 + \frac{1}{2} \)), [and] in chemistry \( \text{CO}_2 \) is produced by adding oxygen to carbon” (Stacey and MacGregor 1994, p. 290). They concluded that students were drawing on notation systems other than algebra thus creating mathematical obstacles. Although conjoining results in a “syntactically well-formed answer” (Stacey and MacGregor 1994, p. 296), this personal shorthand provides invalid answers.

Following the CSMS algebra study, the ‘Strategies and Errors in Secondary Mathematics’ SESM study (Booth 1984), conducted in the UK between 1980-83, aimed to investigate underlying reasons for particular errors. Children took the original CSMS test to identify those who made similar errors to those found by Küchemann (1981). The team then conducted two interviews, six months apart, to hypothesise the causes of observed errors. Reviewing this study Booth (1988) identifies several difficulties. She explains that similar errors were made across age ranges. One misconception identified was the pupils’ ideology of “the focus of algebraic activity and the nature of ‘answers’” (Booth 1988, p. 21). There is a clear correlation between arithmetic and algebra; however the focus of the activity is different. In arithmetic students are taught to find specific numerical answers to set questions, however in algebra the focus is on relationships and expression of generalised rules. Arithmetic paradigm could be one reason why pupils often have the desire to find an exact solution to algebraic problems. Booth (1988) cites an interview with a pupil, Michael, regarding the question “West Ham scored \( x \) goals and Manchester United scored \( y \) goals. What can you write for the number of goals scored altogether?” (Booth 1984, p. 20). Michael felt this could not be answered unless provided with numbers to substitute for \( x \) and \( y \). With input from the interviewer he eventually comes around to the answer “\( x + y \)”, however is unhappy with this solution. He proceeds to express this as just “\( z \)”, showing that some students prefer to provide closed single term solutions which may seem ‘more mathematically correct’.

Booth (1988) also discusses other confusion in unclosed expressions, since an answer such as \( n + 3 \) could be either an instruction (add 3 to \( n \)) or an answer having performed the addition. Similar results were found in higher level algebra (Tirosh and Almog 1989) where students find it difficult to accept complex numbers as numbers. They found that after two months some students regarded \( 3 + 2i \) as ‘not a number’, many describing it as “[containing] operations that still need to be executed” (Tirosh and Almog 1989). A ‘lack of closure’ is not just an issue for secondary school algebra.
In contrast Norton and Cooper (2001) focused on the importance of closure in arithmetic looking at later implications for algebra. Year 9 level students had to answer questions testing their ability in arithmetic before algebraic study and received no supporting material. The question “If I add 56 to the right hand side, how do I keep the equation $139 \times 43 = 5977$ equal?” was answered correctly by few students. It is suggested that, in terms of closure, this question may have been difficult since $139 \times 43$ is too large to mentally calculate and ‘close’, and the question had further been closed as it was set equal 5977. Many students wrote, “I don’t know what to do”, which could be because they expected to provide an answer after the equals sign as in arithmetic.

There are clearly a number of underlying problems in the way that algebra is taught and how students interpret that teaching. One important problem is this ‘lack of closure’ dilemma, driving students to incorrect answers. I think it is important to build upon prior learning experiences of young people, and it is identified by Watson, Jones and Pratt (2013) that relations between quantities is the main link between primary curriculum mathematics and the study of algebra in secondary school.

**Part two: Design of research informed activities to address difficulties**

Activities were conducted in one lesson with a year 7 class. I introduced them to the key concepts of using letters as numbers and collecting like terms, then they tackled my activities. This provided me with the opportunity to see whether students preferred ‘closed’ answers or were happy to give ‘unclosed’ answers.

Most research into ‘lack of closure’ comprises tests and interviews to find misconceptions. Booth (1984) has devised teaching experiments and Swan (2000; 2008) has designed and recommended teaching approaches “to allow students to construct and reflect on meanings for expressions and equations” (Swan 2000, p. 16).

Design-based research in mathematics education is still in its early stages (Swan 2014) and as such we are only beginning to see the “study of learning through systematic design of teaching strategies and tools” (Swan 2014). I reviewed some of this design-based research (Swan 2000; 2008) in order to devise a lesson of activities for my year 7 class tackling some algebraic problems.

My class had had little or no exposure to algebra, therefore it would have made sense to teach the class immediately before progressing into the main activities. Before doing this I felt that providing the class with some of the questions seen in research studies would provide the students with a brief opportunity to explore algebra before being fully aware of what to do. This activity allowed the students to make their mistakes before learning the concepts, and by returning to the activity at the end of the lesson I was able to ensure that they knew how to use letters to represent numbers, collect like terms and were aware of possible misconceptions which arise in this area of mathematics.

I felt it was appropriate to use questions which featured in the Norton and Cooper (2001) arithmetic test and questions from the CSMS study used in the SESM study and reviewed by Booth (1988). The questions I used for discussion at the beginning of the lesson can be seen in Table 1.

Using these questions at the beginning of the lesson provided an opportunity for mistakes so misconceptions in this topic could later be identified. This task identified some difficulties learners have with algebra and the arithmetic preceding it. The questions from Booth’s (1984) SESM study also provided an opportunity to see whether students wish to ‘close’ answers.

For the actual teaching of the key concepts of using letters as numbers and collecting like terms I used explanations from the textbook used in the school’s scheme of work, MathsLinks Student Book 7C (Appleton et al. 2008), since I felt they provided concise, explicit and relevant content.

One activity I considered was an arithmetical activity involving questioning with students such as, ‘7 = 3 + 4, can you suggest an alternative way to make 7?’ This would encourage students to realise there are multiple ways to make one answer and hopefully suggest equations don’t have to be read left to right (e.g. 3 + 4 = 7) as is commonly assumed. It would also tackle the idea that both sides of an equation must be equal, what happens on the left must happen on the right in order for the equals sign to actual mean ‘equal’. I decided against using this activity since it was more arithmetically based and I wanted to observed students exploration with actual algebraic expressions.

Instead the main activity I decided to use was Algebra True or False? (Wright 2012, see appendix A) which is a card sorting task involving algebraic statements with a single variable ‘a’. The students were asked to sort these into categories ‘True’ or ‘False’, similar to an activity designed by Swan (Swan 2000). I felt this was suitable for the class and adapted the activity for use in my lesson by creating additional cards (see appendix B) with algebraic statements involving two variables ‘a’ and ‘b’. This activity provided students with the opportunity to consider whether some actual algebraic statements were in fact valid. They also had the chance to determine whether answers which had been closed, or left unclosed, were ‘True’ or ‘False’, thus addressing the ideas of a ‘lack of closure’.

Whilst reviewing resources suggested on the Nuffield website (Watson et al. 2013) I also decided to implement the Perimeter Expressions (NRich 1997-2014, see appendix C) activity during the lesson. In this task the students were provided with cut outs of rectangles (see appendix D) posed in the problem. One rectangle was labelled as having side lengths ‘a’ and ‘b’. The students were then asked to discuss and were expected to come to agreement that the perimeter of a particular shape was as given in the question. They should have then progressed on to more difficult questions requiring use of algebra to determine perimeters and areas. This activity required mathematical reasoning and also promoted the idea that conjoining terms isn’t possible, since in perimeter we must add together each edge, which should have been clear following their learning in the lesson that ‘ab’ is a multiplication which would then represent area, thus addressing ‘lack of closure’ in algebraic answers. Encouraging collaborative work (Swan 2000; 2008; 2014) also gave the chance for groups to set the challenge for others and provided me with the prospect of listening to students conversations to identify some sort of evidence of understanding (e.g. conversations about simplification or referral to
the perimeter of the shapes, etc.). In this activity students were expected to discuss the use of algebra in representing perimeters and areas, and collectively come to an agreement that conjoining terms is not possible when adding together algebraic terms.

As mentioned earlier, I returned to the opening discussion at the end of the lesson. From the teaching and activities undertaken, students should have reflected upon their initial thinking, the new ideas introduced and the associated discussions. They were then asked to review, discuss and gain understanding of the correct answers.

After my review of the research the structure of my lesson proceeded as follows:

- **Starter:** pair or small group discussion based on questions from research based studies by Norton and Cooper (2001) and Booth (1984).
- **Introduction to algebra:** use of ideas set out in textbook by Appleton *et al.* (2008) introducing the class to using letters as numbers and the idea of collecting ‘like’ terms.
- **Algebra True or False?** (Wright 2012) activity: card sorting activity in pairs or small groups, involving categorising algebraic expressions as ‘True’ or ‘False’.
- **Perimeter Expressions** (NRich 1997-2014) activity: pair or small group discussion of ideas tackling tasks set out in this activity. Recognising the connection between the algebra and the perimeter was the first step before then discussing ideas about how they might solve the remaining questions.
- **Plenary:** brief pair or small group discussion to review answers provided at the beginning of the lesson followed by a class discussion with students providing explanations and reasoning of their answers to the questions provided.

The ideas I selected for my lesson aimed to address some of the mistakes that students make during the study of algebra. They were also to address the misconceptions of the underlying concepts, in particular some of the misuse and understanding of the equals sign and the ‘lack of closure’ which is often provided by some algebraic answers.

**Part three: Conducting, analysing and evaluating the activities**

The lesson began with a discussion activity using questions seen in Table 1. Whilst circulating the classroom, I spoke to pupils who were able to answer question one without difficulty, recalling BIDMAS and working out the brackets before calculating the multiplication. This reflects results seen in the research conducted by Norton and Cooper (2001) who found almost all test students getting the correct answer.

Question two caused more difficulties for some students. Many restated the question during discussions with some also saying, “Add them together to get 60 then divide by 6 to get 10”. These students were reminded by partners that the question asked for an answer without doing the addition first. Other students were prompting their peers to explain ‘how’ when they claimed the answer to be 10, but with the response “I can’t explain it.” Students struggled to find both answer and explanation, required to fully answer the question, similarly reflecting results in Norton and Cooper’s (2001) research, where few students succeeded in answering the question correctly. Eventually I heard one student claim, “I think you divide 36 by 6 first, and then divide 24 by 6 then add them together. That would be 6 and 4 which would make 10.” This student had understood the question requirements, not calculating the addition first, and made an assumption that the division could be done first instead.
This student’s partner was quick to accept this answer having calculated the addition first himself and arriving at the same answer.

Discussion of question three featured as a way to determine students’ interpretation of a question containing potential ‘lack of closure’, and I expected some conjoining of terms to occur. Every student I spoke to claimed to have got the answer easily (8y) with little peer discussion, just agreement with this answer. When I asked for an explanation, most suggested adding the numbers together and putting y at the end. One student was happy with his answer ‘8y’ but asked me, “Sir, if you had ‘Add 3x to 5y’ would it be 3x + 5y?” He had realised that you can’t combine the letters together when adding different variables, but had not also applied this reasoning to adding numbers and letters. This answer is similar to one provided by Peter in his SESM interview (Booth 1988), but I didn’t hear any student make Peter’s claim, “Could be … a yacht. […] Could be yoghurt. Or a yam.” (Booth, 1998, p.27) The students in the class did not associate the letter with any particular object. It is clear, however, that the ‘lack of closure’ is something these students, who are yet to study algebra, are not yet comfortable with – hence their answer 8y.

The final question caused some controversy. More students identified that the answer would be \( x + y \), although some claimed you could put the letters together as \( xy \). I didn’t hear any responses similar to Michael’s in his SESM interview (Booth 1988) – this could be because they haven’t yet explicitly seen algebra, but closure in the final solution was clearly a preferred way to answer the question.

This discussion provided opportunity for the students to discuss their own mathematical interpretations of questions. It was clear that individuals were learning from their peers, refuting incorrect suggestions made with explanations of alternative answers. Despite not all of the correct answers being found by all groups at this stage, it provided the chance for the students to discuss possible reasoning behind their own answers.

I then introduced the key concepts of using letters as numbers and collecting like terms. Using the textbook MathsLinks Student Book 7C (Appleton et al. 2008) I began with use of letters as numbers, explaining basic notation and showing examples with answers provided by the class. They then tackled three questions based on notation before answers were revealed and explained by individuals. This process was repeated with collection of like terms. Those who provided answers seemed to have grasped the ideas, providing coherent explanations of how they found the answer. For example, a student explained for the question, “Given that \( m = 4 \) and \( n = -3 \), find the value of \( m(n - 2) \),” that, “If you take a negative from a negative it just gets bigger in the negative direction, so \(-3 - 2\) is \(-5\), and then you multiply it by 4 which would be \(-20\).” I didn’t mention use of brackets in algebraic expressions during my introduction as I wanted to see what assumptions would be made in this sort of question. This student correctly assumed multiplication for the expression, which shows that he was able to interpret an algebraic expression from the knowledge he now had about algebraic notation.

The first activity the students tackled was Algebra True or False? (Wright 2012). Students were provided with the cards and Additional Cards for Algebra True or False? (self-designed) containing algebraic expressions and were asked to sort them into a table with columns ‘True’ and ‘False’ – this allowed students to change their minds during the activity. The students were asked to discuss the statements in twos or threes to determine whether the statements were true or false. All the students I talked to and observed seemed comfortable with this task, discussing like terms, difference in letters and adding numbers to letters. Interestingly those students who had claimed “Add 3 to 5y” was equal
to ‘8y’ did not seem to have difficulty in noticing that, for example, the card “4a + 1 = 5a” was in fact false, some stating “You just can’t add one to 4a, because it has no a so you can’t make 5a because one would need to be a, so it’s got to stay the same.” The activity allowed some misconceptions already seen to resurface in some discussions, which was a similar outcome to what Swan (2000) wanted from his ‘Always’, ‘Sometimes’, ‘Never’ True activity. When I was confident most students had completed the activity and their discussions I asked the groups to stick their cards under the headings. I then asked students to identify which category some of the cards should have gone in, and to provide an explanation. An explanation for ‘3a + 2b = 5ab’ was “Well ‘a’ and ‘b’ are not like terms so you can’t just join them, it needs to stay as ‘3a + 2b’.” This student had taken into account the teachings prior to the activity and had identified that each letter represents something different and cannot be collected into one term. This activity helped the students to become familiar with algebraic notation and encouraged discussion about collection of like terms and conjoining of terms. They were able to develop their ideas of algebraic notation from the earlier teachings and discuss the validity of algebraic statements in relation to this.

The other activity was the Perimeter Expressions (NRich 1997-2014) activity, however due to limited time it was not fully implemented and the results do not show the true potential of the classes’ ability to tackle the activity, so are not described here.

To conclude, I returned to the discussion questions from the beginning of the lesson. I gave the class extra time to review their original answers before asking for contributions. Upon request for the answer to question one, almost every hand went up and the answer ‘21’ was given. The explanation, “Well it’s BIDMAS so you need to do the brackets first, so 2 + 1 = 3 and then you times by 7 which is 21 which is the answer,” was given and the class agreed. I don’t think anyone had a different answer or reasoning and everyone seemed to be happy and comfortable with this.

For question two the one pupil said, “You just can’t do it. You have to add the numbers together first otherwise you won’t get the right answer.” Other students in the class agreed with this response, as in Norton and Cooper’s (2001) research; however a small number of students put their hands up claiming there was a way. One of these explanations was, “Well if you add them together first and then divide by 6 the answer will be 10. You can do it without though, if you take 36 and divide that by 6 first that is 6. Then if you divide 24 by 6 you get 4. Add those together you get 10 as well which is the same answer, so that’s the answer?” The student who provided the answer did not seem overly confident it was right as he provided it, though the class began to agree that this was how you could do the question. With further similar examples this learning could be reinforced in order to increase confidence in this approach.

Question three had caused errors at the beginning of the lesson and was answered with the explanation, “Well originally I did 3 add 5 which is 8 and then just added the y on the end which is 8y… But that’s not right. Because of like terms. The terms aren’t like so if you add 3 to 5y it’s just going to be 5y + 3? That is the answer.” This student had provided an explanation of his original incorrect answer however had noticed its invalidity identifying from ideas learnt in the lesson that his answer was wrong. He was then able to explain confidently what the correct answer would be using these facts. Most of the class agreed that ‘8y’ was incorrect and this new explanation using like terms made more mathematical sense. This student had single-handedly convinced the rest of his classmates how to approach this question.
For the final question two students attempted explanations. The first claimed, “Well you have got $x$ goals and $y$ goals so the total would be $xy$ goals.” The second student said, “Well you can’t just join them together. You need to add them, if West Ham had scored 2 goals and Manchester United had scored 3 goals then the total is $2 + 3$ goals, which is 5. So the answer to the question would be $x + y$ because we don’t know what $x$ or $y$ are.” The second student had clearly gained a good idea about algebraic operations and was happy that collecting like terms is the only way to simplify ‘unclosed’ expressions. No one suggested simplifying the expression to $z$ as seen by Booth (1988), but it was clear that only a handful of the students had fully grasped the ideas of algebraic ‘lack of closure’.

At the beginning of the lesson students interpreted algebraic questions with a systematic approach leading to conjoining of terms, possibly drawing from ideas seen in other areas as suggested by Stacey and MacGregor (1994) – especially since they studied fractions during the preceding lesson. Having seen how algebraic notation works from explanation and examples, most students were then able to collaboratively determine the validity of algebraic statements. Having used Swan’s (2000; 2008; 2014) idea of using rich, collaborative tasks I was able to encourage discussion in small groups which allowed some conflict and explanations of answers.

From this experience I have learnt not to rush tasks, and not to try and fit too much into the time otherwise not all the learning points can be discussed. I could have extended the Algebra True or False? (Wright 2012) activity to include more difficult expressions, or developed the activity into an ‘Always’, ‘Sometimes’, ‘Never’ True idea as used by Swan (2000). In changing the task into an ‘Always’, ‘Sometimes’, ‘Never’ True activity students would be able to consider expressions for longer and discuss their ideas in more depth – this would have further developed their understanding of the concepts they were learning about. I believe introducing a second, very different, activity (Perimeter Expressions) did not allow the students to progress far enough into the full understanding that could be gained from the card activity.

I believe that the students were able to grasp an understanding of why ‘lack of closure’ occurs and have gained some confidence in providing answers which are ‘unfinished’ having identified that expressions can only be simplified if there are ‘like terms’ involved.

Conclusion

The research conducted in the study of algebra is extensive and has identified many difficulties, as discussed in part one. There are a significant number of academics who have focused on this idea of a ‘lack of closure’ in algebraic answers. Of these researchers, Malcolm Swan has provided designs for activities which try to combat the misconceptions which have been identified through earlier research. Conducting a selection of activities with a class who was yet to study algebra provided me with valuable insight to some of the interpretations students make before actually being exposed to the correct notation used in algebra. It is clear that closure of answers was preferred by the students in my class at the beginning of the lesson, but after being introduced to the starting concepts and algebraic notation the majority of the class were able to provide answers which had a ‘lack of closure’, being confident that they couldn’t simplify their expressions further due to there being no ‘like terms’ remaining in the expression.

References


Appendix A: *Algebra True or False?* (Wright 2012)

**Algebra True or False?**
Sort the statements below into those which are true and those which are false. Then explain how you know.

<table>
<thead>
<tr>
<th>7a + 2 = 2 + 7a</th>
<th>a + a = 2a</th>
<th>2a - a = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a + 3a = 5a</td>
<td>2a x 3a = 5a</td>
<td>5a - 3a = 2a</td>
</tr>
<tr>
<td>4a + 1 = 5a</td>
<td>a + a + a = 3a</td>
<td>4 x a = 4a</td>
</tr>
</tbody>
</table>

Appendix B: Additional cards to be used with the *Algebra True or False?* (Wright 2012) activity (Self-designed)

**Additional Algebra True or False? Cards**
To be used alongside *Algebra True or False?* (Wright 2012)

<table>
<thead>
<tr>
<th>3a + 2b = 5ab</th>
<th>a + 3b = 3b + a</th>
<th>5a x b = 5ab</th>
</tr>
</thead>
<tbody>
<tr>
<td>a + a + b = 2a + b</td>
<td>a + b + b = 2ab</td>
<td>2a - b = b - 2a</td>
</tr>
</tbody>
</table>
Appendix C: *Perimeter Expressions* (NRich 1997-2014)

**Perimeter Expressions**

**Stage: 3 ★**

Charlie took a sheet of paper and cut it in half.

Then he cut one of those pieces in half, and repeated until he had five pieces altogether.

He labelled the sides of the smallest rectangle, $a$ for the shorter side and $b$ for the longer side.

Here is a shape that Charlie made by combining the largest and smallest rectangles:

Check you agree that the perimeter is $10a+4b$.

Alison combined the largest and smallest rectangles in a different way. Her shape had perimeter $8a+6b$. Can you find how she might have done it?

Charlie and Alison made sure their rectangles always met along an edge, with corners touching.

Can you combine the largest and smallest rectangles in this way to create other perimeters?

Create some other shapes by combining two or more rectangles, making sure they meet edge to edge and corner to corner. What can you say about the areas and perimeters of the shapes you can make?

*If you have a friend to work with, you could each create a shape and work out the area and perimeter. Can you recreate each other's arrangement if you only*
know the area and perimeter?

**Here are some questions to consider:**

What’s the largest perimeter you can make using ALL the pieces?

Can you make two different shapes which have the same area and perimeter as each other?

Can you make two different shapes which have the same perimeter but different areas?

How do you combine any set of rectangles to create the largest possible perimeter?

Charlie thinks he has found a shape with the perimeter $7a+4b$. Can you find his shape?

What can you say about the perimeters it is possible to make, if $a$ and $b$ are the dimensions of one of the other rectangles?

*This problem is based on an idea shared by Sue Southward.*

Appendix D: Paper rectangles for use during *Perimeter Expressions* (NRich 1997-2014) activity